Expectation Values of Local Fields in an Integrable Theory after a Quantum Quench

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The expectation values of local fields of any interacting quantum theory after a quench process are key quantities for matching theoretical and experimental results. For quantum integrable field theories, we argue that they can be obtained by a generalization of the Leclair-Mussardo formula and a Bethe Ansatz result of Caux and Konik. Specializing to the Sinh-Gordon model and taking the non-relativistic limit, one can recover the results of Kormos et al. for the Lieb-Liniger model.

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The aim of this paper is to set up a formula that allows the computation of the asymptotic time Expectation Value (EV) of local fields of a $(1+1)$ dimensional Quantum Integrable Field Theory (QIFT) after a quantum quench, i.e. an abrupt change of the parameters of the Hamiltonian. The subject of quantum quenches have recently attracted a lot of attention, both from experimental and theoretical point of view, see for instance [1–18]. QIFT's are special quantum many-body systems since they have an infinite number of conservation laws and their dynamics is strongly constrained (see, e.g. [19] and references therein): their asymptotic time regime is expected to violate the ergodicity property although it may be described by a Generalized Gibbs Ensemble (GGE), as advocated in [14] and shown in a series of papers, among which [15–17]. The many questions coming from this general context (e.g. the setting of equilibration without a proper thermalization[1], the importance of rare states [12, 13], the phenomena of pre-thermalization [8–11], the role of the local as well as the non-local charges and the identification of proper density matrix [16], the relationship between pre-quench and post-quench operators [18], the hints coming from Bethe Ansatz [15, 26, 27], etc.) have stimulated an intense research activity and a lot of progress has been made on many of these topics. However, despite these important advances, it seems that a proper formula for computing these asymptotic EV in QIFT is still missing in the literature. This formula is proposed here and illustrated with the avail of the Lieb-Liniger model, i.e. the non-relativistic limit of the Sinh-Gordon (ShG) theory [23]. It is also worth to mention that, if one restores a $\hbar$ dependence, the limit $\hbar \to 0$ of the formula given below identifies the EV in the purely classical dynamics of relativistic integrable models [25].

In a nutshell, the formula is just a generalization of the LeClair-Mussardo formula previously obtained in the context of QIFT at a finite temperature [20]. Namely, let $\mathcal{O}(x,t)$ be the local field and $\langle\psi_0|\mathcal{O}(t)|\psi_0\rangle$ its expectation value on a macroscopic state $|\psi_0\rangle$ of the theory (i.e. made of an infinite superposition of multi-particle states), which we assume to be translation invariant and not an eigenstate of the Hamiltonian. For simplicity we will consider a QIFT made of only one type of particle, the generalization to more general case being straightforward. Let's define the Dynamical Average (DA) of this field on $|\psi_0\rangle$ as the time average after the quench at $t=0$

$$\langle\mathcal{O}\rangle_{DA} \,\equiv\, \lim_{t,\to\infty} \int_0^t dt\; \langle\psi_0|\mathcal{O}(t)|\psi_0\rangle \tag{1}$$

Let's instead define the Ensemble Average (EA) of this observable on this state as

$$\langle\mathcal{O}\rangle_{EA} \,\equiv\, \sum_{n=0}^{\infty}\frac{1}{n!}\int_{-\infty}^{\infty}\left(\prod_{i=1}^{n}\frac{d\theta_i}{2\pi}f(\theta_i)\right)\langle\overleftarrow{\theta}\,|\mathcal{O}(0)|\,\overrightarrow{\theta}\rangle_{\rm conn}\,, \tag{2}$$

where $f(\theta_i) = (e^{\varepsilon(\theta_i)} - S(0))^{-1}$ (with $S(\theta)$ the exact two-body S-matrix of the model), while $|\overrightarrow{\theta}\rangle \equiv |\theta_1,\ldots,\theta_n\rangle$ ($\langle\overleftarrow{\theta}| \equiv \langle\theta_n,\ldots,\theta_1|$) denotes the asymptotic states of the integrable theory expressed in terms of the rapidities θ_i, with $E(\theta) = m\cosh\theta$, $p(\theta) = m\sinh\theta$. This formula employs both the pseudo-energy $\epsilon(\theta)$, solution of a Generalized Bethe Ansatz (GBA) equation based on the state $|\psi_0\rangle$ [27], and the connected diagonal Form Factor (FF) of the operator $\mathcal{O}$, defined as $\langle\overleftarrow{\theta}|\mathcal{O}|\overrightarrow{\theta}\rangle_{\rm conn} = FP\left(\lim_{\eta_i\to 0}\langle 0|\mathcal{O}|\overrightarrow{\theta},\overleftarrow{\theta} - i\pi + i\overleftarrow{\eta}\rangle\right)$ where $\overleftarrow{\eta} \equiv \eta_n,\ldots,\eta_1$ and FP in front of the expression means taking its finite part, i.e. omitting all the terms of the form η_i/η_j and $1/\eta_i^p$ where p is a positive integer. Our claim is that it holds the following equality between the two averages

$$\langle\mathcal{O}\rangle_{DA} \,=\, \langle\mathcal{O}\rangle_{EA}\;\;. \tag{3}$$

This equality provides an efficient way of computing the EV in a QIFT in terms of a generically very fast convergent series. As shown below, the Ensemble Average of eq.(2) is nothing else but the formula that allows us to extract the *regularized finite* expression of the EV coming from the Diagonal Ensemble, reached by using the Dynamical Average of eq.(1).

To spell out the content of (3), let's start by showing the computation of the EV of local fields in the simplest QIFT, i.e. the free theory. In the infinite volume, the

solution of the eq. of motion $(\Box + m^2)\,\phi(x,t) = 0$ is

$$\phi(x,t) = \int_{-\infty}^{\infty} \frac{d\theta}{2\pi} \left[A(\theta)\, e^{-i(E(\theta)t - p(\theta)x)} + c.c. \right] \quad (4)$$

The modes $A(\theta)$ and $A^\dagger(\theta)$ are fixed in terms of the boundary conditions at $t = 0$, i.e. by the expectation value of the field and its time derivative on the states $|\psi_0\rangle$, and satisfy the commutation relation $[A(\theta), A^\dagger(\theta')] = 2\pi\delta(\theta - \theta')$. The dynamics of the model is supported by the infinite number of *non-local* conserved quantities given by the mode number occupations $N(\theta) = \frac{1}{2\pi}|A(\theta)|^2$ for any θ. To find the infinite set of *local* conserved quantities it is convenient to go in the light-cone coordinates $\tau = t + x$ and $\sigma = x - t$, where the eq. of motion becomes $\phi_{\sigma\tau} = m^2\,\phi$ and, as a consequence, there is the infinite chain of conservation laws ($n = 1, 2, \cdots$): $\partial_\tau \phi_{n\sigma}^2 = m^2\,\partial_\sigma \phi_{(n-1)\sigma}^2$ and $\partial_\sigma \phi_{n\tau}^2 = m^2\,\partial_\tau \phi_{(n-1)\tau}^2$, where $\phi_{n\sigma} = \partial_\sigma^n \phi$ and analogously for $\phi_{n\tau}$. These equations are of the general form $\partial_\tau A = \partial_\sigma B$ and, going back to the coordinates (x,t), they become the continuity equation $\partial_t(A + B) = \partial_x(B - A)$. One can easily identify the even and odd charges $\mathcal{E}_{2n-1}$ and $\mathcal{O}_{2n-1}$ (the first ones are the energy and momentum of the field): up to normalization, they can be written as

$$\begin{aligned}
\mathcal{E}_{2n-1} &= m^{2n-1} \int \frac{d\theta}{2\pi}\,|A(\theta)|^2\,\cosh[(2n-1)\theta] \\
\mathcal{O}_{2n-1} &= m^{2n-1} \int \frac{d\theta}{2\pi}\,|A(\theta)|^2\,\sinh[(2n-1)\theta]
\end{aligned}$$

They imply that each particle state $|\theta\rangle$ (as well as all multi-particle states) is a common eigenvectors of all these conserved quantities, with eigenvalues

$$\begin{aligned}
\mathcal{E}_n\,|\theta\rangle &= m^{2n-1}\,\cosh[(2n-1)\theta]\,|\theta\rangle \\
\mathcal{O}_n\,|\theta\rangle &= m^{2n-1}\,\sinh[(2n-1)\theta]\,|\theta\rangle
\end{aligned} \quad (5)$$

It is pretty evident that the knowledge of the mode occupation $|A(\theta)|^2$ fixes all the local charges but, under both general mathematical and physical conditions, it is also true the viceversa, alias the knowledge of $\mathcal{E}_{2n-1}$'s and $\mathcal{O}_{2n-1}$'s is enough to fix $|A(\theta)|^2$. Being linearly related one to the other, the two types of conservation laws are then essentially interchangeable. It must be stressed that eq. (5) holds also in interacting QIFT as the ShG model, for instance, where the only substitution is $|A(\theta)|^2$ with the proper action variable $P(\theta)$ in (5).

In the free theory, the exact solution (4) of the eq. of motion allows us to easily compute the stationary EV of any local function $F[\phi(x,t)]$, defined with a proper normal-ordering of the operators. Since we are interested in field configurations with *finite energy density*, our theory has to be defined on a circle of length L and then send $L \to \infty$ so that E/L is always finite, even in this limit. The momenta of the particles will be quantized in unit of $2\pi/L$ which become dense when $L \to \infty$. The

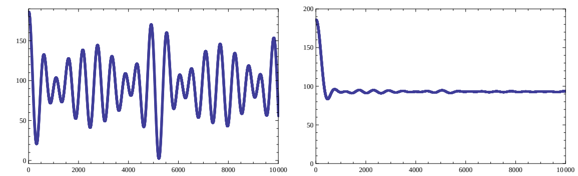

FIG. 1: On the left: time evolution of $\langle\psi_0|:\phi^2(t):|\psi_0\rangle$ for $t = [0, 10^4 t_0]$ and a typical translation invariant state $|\psi_0\rangle$. On the right: its time average.

Dynamical Average of eq.(1) smooths out all microscopic fluctuations of the observables and leads, asymptotically, to stationary values. One way to visualize these EV is to consider a quench in the free theory where the mode occupations are very large, so that the theory can be treated classically and solved in this way [24, 25]: the typical time evolution of a space-average observable $\tilde{F}[\phi(t)]$ is shown on the left plot of Figure (1) where, as time goes by, there are persistent fluctuations. These can be flatten by taking the time average: the mean then rapidly converges to the asymptotic value (not necessarily a thermal one).

Let's consider a quench protocol whose instances have in common just the same energy density E/L. Since the energy is a very degenerate observable, each quench process corresponds to different values of all other conserved charges and the typical result is shown in Figure 2, where one can see a large spread of these EV. This strong dependence on the initial data is easily explained. Focus, for simplicity, the attention on the EV of the observables $:\phi^k(x,t):$ of a free theory. While $\langle :\phi^{2n+1}:\rangle_{DA}$ vanish by symmetry, a phase-stationary argument for the even ones leads to

$$\langle :\phi^2:\rangle_{DA} = \int \frac{d\theta}{2\pi}\,|A(\theta)|^2 \equiv b\ , \quad (6)$$

$$\langle :\phi^{2n}:\rangle_{DA} = (2n-1)!!\,b^n \quad (7)$$

Since they explicitly depend on the initial condition through the $|A(\theta)|^2$'s, they can never be derived by a Gibbs Ensemble average

$$\langle :\phi^{2n}:\rangle_{DA} \neq Z^{-1}\,\mathrm{Tr}\,(:\phi^{2n}:\,e^{-\beta H})\,. \quad (8)$$

Notice, however, that the VE (6) and (7) have precisely the form of the Ensemble Average, since the non-

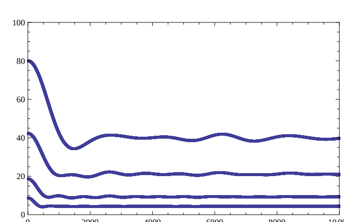

FIG. 2: Time average of $\langle\psi_0|:\phi^2:(t)|\psi_0\rangle$ for different initial states with the same value of the energy density (in this case $E/L = 100$).

zero FF are $\langle\overleftarrow{\theta_m}| : \phi^{2n}(0) : |\overrightarrow{\theta_m}\rangle_{\rm conn} = 2^n(2n)!\delta_{n,m}$ while the pseudo-energy is given by $\epsilon(\theta) = \log(1 + |A(\beta)|^{-2})$, easily derivable for the free theory (with an S-matrix $S(\theta) = 1$) by using the GGE in the form $\rho_{GGE} = Z^{-1}\exp\left[-\int \frac{d\theta}{2\pi}\epsilon(\theta)\,|A(\theta)|^2\right]$. The lagrangian multipliers $\epsilon(\theta)$ (one for each mode) are fixed by the initial occupation numbers as $|A(\theta)|^2 = (e^{\epsilon(\theta)} - 1)^{-1}$ [5]. The statistical weight given by the GGE is equivalent to $\langle A(\theta)A^\dagger(\theta')\rangle_{EA} = 2\pi\delta(\theta-\theta')$, $\langle A(\theta)A(\theta')\rangle_{EA} = \langle A^\dagger(\theta)A^\dagger(\theta')\rangle_{EA} = 0$, from which one can easily get the generating function

$$\langle \exp[i\alpha\phi]\rangle_{EA} \,=\, \exp\left[-\frac{\alpha^2}{2}\int\frac{d\theta}{2\pi}|A(\theta)|^2\right] \quad . \qquad (9)$$

Expanding in power series in α the left/right hand sides and comparing equal powers in α, one recovers the previous results (6), (7) . Notice that, while the EV of various observables explicitly depend on the initial conditions, for the free theory there are however some ratios completely independent from them as, for instance, $\langle\phi^{2n}\rangle/\langle\phi^2\rangle^n = (2n-1)!!$, or $\frac{\log\langle\cos(\alpha\phi)\rangle}{\log\langle\cos(\gamma\phi)\rangle} = -\left(\frac{\alpha}{\gamma}\right)^2$. Thus any violation of the values of these ratios can be attributed to interaction, for the theory under study.

Consider now a translation invariant initial state $|\psi_0\rangle$ and as a post-quench Hamiltonian H the one of an interacting QIFT, with a two-body S-matrix $S(\theta)$. Expanding on the basis of the common eigenvectors of H and all higher charges, its general form is $|\psi_0\rangle = \sum_{p,n} c_{p,n}|\theta_{1,p}\ldots\theta_{n,p}\rangle$, with $P_p = m\sum_i \sinh\theta_{i,p} = 0$. Posing $E_p = m\sum_{i=1}^n\cosh\theta_{i,p}$, at any later time t the EV of a local observable $\mathcal{O}(x,t)$ on $|\psi_0\rangle$ is

$$\frac{\langle\psi_0|\mathcal{O}(t)|\psi_0\rangle}{\langle\psi_0|\psi_0\rangle} = Z^{-1}\sum_{p,q,m,n} e^{-it(E_p-E_q)} \\ \times\; c^*_{q,m}c_{p,n}\langle\theta_{m,q}\ldots\theta_{1,q}|\mathcal{O}(0)||\theta_{1,p}\ldots\theta_{n,p}\rangle$$

($Z = \langle\psi_0|\psi_0\rangle$). Taking the Dynamical Average, one ends up to the Diagonal Ensemble

$$\langle\mathcal{O}\rangle_{DA} \,=\, Z^{-1}\sum_{q,n}|c_{q,n}|^2\langle\theta_{n,q}\ldots\theta_{1,q}|\mathcal{O}(0)||\theta_{1,q}\ldots\theta_{n,q}\rangle$$

As it stands, however, this expression is highly problematic: all terms of the sum in the numerator as well as those present in Z are in fact *divergent*. Concerning those in Z, the divergencies come from the normalization of the eigenstates, $\langle\theta'_m,\ldots\theta'_1|\theta_1\ldots\theta_n\rangle = \prod_i\delta(\theta'-\theta)$, which gives rise to $[\delta(0)]^n$ when $\theta'_i = \theta$. Concerning the numerator, the divergencies come from the Form Factor $\langle\theta'_m\ldots\theta'_1|\mathcal{O}(0)||\theta_1\ldots\theta_n\rangle$, once evaluated at $\theta'_i = \theta_i$. These divergencies are an unavoidable consequence of the kinematical pole structure of the Form Factors [28].

Luckily enough, the cure of this problem has been already found by LeClair and Mussardo in the context of finite-temperature correlators [20], where one encounters a similar difficulty: the remedy consists in employing the Finite Part of the Form Factors (which are regular functions of the rapidities) and the pseudo-rapidity $\epsilon(\theta)$ (solution of a Bethe Ansatz equation and regular function as well). All these steps are implemented by defining the theory on a finite length L and then finally send $L\to\infty$. In summary, the finite and regularized expression that has to be associated to the Diagonal Ensemble is the one given in eq.(2).

For the quench situation in a QIFT, the only question left is then how to find the explicit expression of the pseudo-rapidity $\epsilon(\theta)$. This problem that has been recently solved by Caux and Konik [27]. We have to remind that on an interval L, each multi-particle state is in correspondence with a sequence of relative integers $\{N_1,\ldots,N_n\}$ (all different and here taken to be symmetric with respect to 0) entering the Bethe Ansatz equations [21, 22]

$$mL\sinh\theta_i + \sum_{k\neq i}\varsigma(\theta_i-\theta_k) \,=\, 2\pi N_i \quad , \qquad (10)$$

where $\varsigma(\theta) = -i\log S(\theta)$ is the phase-shift. Given the N_i's, the rapidites solution of these equations (the *roots*) are uniquely fixed and, for $L\to\infty$, they form a distribution $\rho^{(r)}(\theta)$. This function, in turns, determines through a linear integral equation the distribution $\rho^{(h)}(\theta)$ of the *holes*, i.e. the solutions of eq.(10) corresponding to a skipped sequence of N_i's, since [21, 22]

$$\rho^{(h)}(\theta) = -\rho^{(r)}(\theta) + \frac{mL}{2\pi}\cosh\theta + \int\frac{d\theta'}{2\pi}\varphi(\theta-\theta')\rho^{(r)}(\theta') \qquad (11)$$

where $\varphi(\theta) = d\varsigma(\theta)/d\theta$. If one knew both $\rho^{(r)}(\theta)$ and $\rho^{(h)}(\theta)$, one could have access to $\epsilon(\theta)$, since this quantity is given by

$$\epsilon(\theta) \,=\, \log\frac{\rho^{(h)}(\theta)}{\rho^{(r)}(\theta)}\,. \qquad (12)$$

In the thermodynamic limit there is a large number of quantum states compatible with the densities $\rho^{(r)}$ and $\rho^{(h)}$, so there is a macroscopic entropy $S[\rho^{(r)},\rho^{(h)}]$. Usually, the determination of $\rho^{(r)}$ and $\rho^{(h)}$ passes through the minimization procedure of the Free Energy [21, 22], a functional that in a QIFT involves all conserved charges [15, 26]. To fix the relative lagrangian multipliers may be not an easy task but, as shown in [27], one can actually bypass this problem and avoid altogether to follow the usual procedure adopting instead this plan of actions: (i) denoting by $\theta_{p,i}$ the solution of eq.(10) relative to the generic multi-particle state $|\theta_{1,p}\ldots\theta_{n,p}\rangle$, associate to it the distribution $\rho_{p,n}(\theta) = \sum_i^n\delta(\theta-\theta_{p,i})$ and take as density of the roots of the initial state $|\psi_0\rangle$ the weighted sum of all these distributions $\rho^{(r)}(\theta) = \sum_p|c_{p,n}|^2\rho_{p,n}(\theta)$; (ii) substitute this $\rho^{(r)}$ into eq.(11) and solve for $\rho^{(h)}(\theta)$ to get then $\epsilon(\theta)$ from eq.(12). For the Generalized Free Energy one has $F = -\frac{mL}{2\pi}\int d\theta\cosh\theta\log(1+e^{-\epsilon(\theta)})$, while for the EV of the conserved charges on $|\psi_0\rangle$ [26]

$$\mathcal{E}_{2n-1} \,=\, \frac{mL}{2\pi}\int d\theta\cosh\theta\, f(\theta)\frac{\partial\epsilon(\theta)}{\partial\alpha_{2n-1}}$$

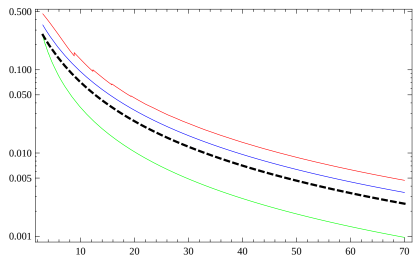

FIG. 3: Quench instances of $\langle(:\psi^{\dagger 2}\psi^2:)$ versus the dimensionless coupling constant γ of the Lieb-Liniger model. The thermal equilibrium curve is the dashed one.

where

$$f(\theta) = \frac{1}{e^{\epsilon(\theta)} + 1} \tag{13}$$

is the filling fraction of the states, while $\partial\epsilon/\partial\alpha_{2n-1}$ is the solution of the *linear* integral equation

$$\frac{\partial\epsilon(\theta)}{\partial\alpha_{2n-1}} = m^{2n-1}\cosh[(2n-1)\theta] + \int \varphi(\theta-\theta') f(\theta') \frac{\partial\epsilon(\theta')}{\partial\alpha_{2n-1}}$$

A nice check of eq.(3) is provided by the Lieb-Liniger model regarded as non-relativistic limit of the ShG model [23]. Using the Form Factors of this theory, one recovers the formulas obtained in [29] for the EV of the local composite fields $:\psi^{\dagger k}(x)\psi^k(x):$ of the Lieb-Liniger model, where the function $f(p)$ employed in [29] is nothing else but the same $f(\theta)$ of eq.(13). As an example, Fig. 3 shows the EV of $:\psi^{\dagger 2}(x)\psi^2(x):$ versus γ, the dimensionless coupling of the Lieb-Liniger model, for different quench processes (this is the equivalent of Fig.2 for an interacting case): no surprise, there is a spread of values of this quantity, since there are of course innumerable many ways to get out of equilibrium (and QIFT keeps memory of it) but only one way to be at equilibrium.

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Note Added. There has been recently another proposal to compute the EV [30], based on the Bethe Ansatz and checked for the free case of the quantum Ising model. Although very similar to mine, it is not clear to me that the two proposals are equivalent and further analysis is needed to prove or disprove their identity.

[1] T. Kinoshita, T. Wenger, D. S. Weiss, Nature 440, 900 (2006); M. Greiner, O. Mandel, T. W. Hänsch, and I. Bloch, Nature 419 51 (2002); S. Hofferberth, I. Lesanovsky, B. Fischer, T. Schumm, and J. Schmiedmayer, Nature 449, 324 (2007).
[2] S. Trotzky Y.A. Chen, A. Flesch, I. P. McCulloch, U. Schollwöck, J. Eisert, I. Bloch, Nature Phys. 8, 325 (2012).
[3] M. Cheneau, P. Barmettler, D. Poletti, M. Endres, P. Schauss, T. Fukuhara, C. Gross, I. Bloch, C. Kollath, and S. Kuhr, Nature 481, 484 (2012).
[4] J.M. Deutsch, Phys. Rev. A 43, 2046 (1991); M. Srednicki, Phys. Rev. E 50, 888 (1994).
[5] P. Calabrese and J. Cardy, J. Stat. Mech. P06008 (2007)
[6] A. Polkovnikov, K. Sengupta, A. Silva and M. Vengalattore, Rev. Mod. Phys. **83**, 863-883 (2011) and references therein.
[7] A. Iucci and M. A. Cazalilla, Phys. Rev. A 80, 063619 (2009); T. Barthel, U. Schollwöck, Phys.Rev.Lett.100, 100601 (2008); S. R. Manmana, S. Wessel, R. M. Noack, A. Muramatsu, Phys. Rev. B 79, 155104 (2009); P. Barmettler, M. Punk, V. Gritsev, E. Demler, and E. Altman, New J. Phys. 12, 055017 (2010); D. Rossini, A. Silva, G. Mussardo, G. Santoro, Phys. Rev. Lett. 102, 127204 (2009); D. Rossini, S. Suzuki, G. Mussardo, G. E. Santoro, A. Silva, Phys. Rev. B 82, 144302 (2010)
[8] J. Berges, S. Borsanyi, C. Wetterich, Phys.Rev.Lett. 93 (2004) 142002.
[9] M. Kollar, F. A. Wolf, M. Eckstein, Phys. Rev. B 84, 054304 (2011)
[10] M. C. Banũls, J. I. Cirac, and M. B. Hastings, Phys. Rev. Lett. 106, 050405 (2011).
[11] T. Kitagawa, A. Imambekov, J. Schmiedmayer, E. Demler, New J. Phys. 13 (2011) 073018
[12] G. Biroli, C. Kollath, and A.M. Läuchli, Phys. Rev. Lett. 105, 250401 (2010).
[13] G. P. Brandino, A. De Luca, R.M. Konik, G. Mussardo, Phys. Rev. B 85, 214435.
[14] M. Rigol, V. Dunjko, V. Yurovsky, and M. Olshanii, Phys. Rev. Lett. 98, 050405 (2007); M. Rigol, V. Dunjko, M. Olshanii, Nature **452**, 854 (2008).
[15] D. Fioretto, G. Mussardo, New J. Phys. 12, 055015 (2010).
[16] P. Calabrese, F.H.L. Essler, and M. Fagotti, Phys. Rev. Lett. 106, 227203 (2011); J. Stat. Mech. P07016 (2012); J. Stat. Mech. P07022 (2012).
[17] V. Gurarie, J. Stat. Mech. (2013) P02014; T. Caneva, E. Canovi, D. Rossini, G. E. Santoro, A. Silva, J. Stat. Mech. (2011) P07015
[18] S. Sotiriadis, D. Fioretto, and G. Mussardo, J. Stat. Mech. (2012) P02017.
[19] G. Mussardo, Statistical Field Theory. Oxford Univ. Press, Oxford (2010).
[20] A. LeClair, G. Mussardo, Nucl. Phys. B 552, 624 (1999).
[21] C.N. Yang, C.P. Yang, J. Math. Phys. **10**, 1115 (1969).
[22] Al.B. Zamolodchikov, Nucl. Phys. B 342 (1990), 695.
[23] M. Kormos, G. Mussardo, A. Trombettoni, Phys. Rev. A **81**, 043606 (2010).
[24] D. Boyanovsky, C. Destri and H.J. de Vega, Phys. Rev. D **69**, 045003 (2004).
[25] G. Mussardo, P.E. Assis, A. De Luca, in preparation.
[26] J. Mossel, J.S. Caux, J. Phys. A: 45, 255001 (2012).
[27] J.S. Caux, R. Konik, Phys. Rev. Lett. 109, 175301 (2012).
[28] F. A. Smirnov, Form Factors in Completely Integrable Models of Quantum Field Theory (World Scientific, Singapore, 1992).
[29] M. Kormos, Y. Z. Chou, A. Imambekov, Phys. Rev. Lett. 107, 230405 (2011).
[30] J.S. Caux and F. Essler, arXiv:1301.3806.